CHAPTER 15

Towards the universal characteristic

1 Leibniz and the universal characteristic

It is not uncommon to see formalists refer to Leibniz as the originator of the *reasoning piano* of Stanley Jevons: the machine that will grind out the solutions to all mathematical questions – just like the machine that will grind out every book whatsoever. They claim that this is what Leibniz's *unversal characteristic* is. This view is immediately refuted by glancing at the works of Leibniz. It is also an example of anachronistic thinking – the attribution of conceptions arising late in the history of ideas to eminent thinkers of the past. In his introduction to Leibniz's logical papers Parkinson writes: -

> In *De Arte Combinatoria* (1666) Leibniz distinguishes the logic of invention from the logic of judgement. Leibniz introduces the idea of an alphabet of human thoughts. "He states the subject/predicate distinction. He next proposes, as a task for inventive logic, the problem of determining all the possible predicates of any given subject, and all the possible subjects of any given predicate." A term is either a subject or predicate. "He is clearly using 'term' in the traditional sense of the subject or predicate of a proposition, and the fact that he speaks in this context of an alphabet of human *thoughts* indicates that he regards such terms as concepts. The analysis, then, is one of concepts; stated roughly, Leibniz's view is that every concept is either ultimate and indefinable, or is composed of such concepts. The indefinable concepts are called by Leibniz 'first terms', and a list of these constitutes what he was later to call the 'alphabet of human thoughts', for derivative concepts are formed from first terms in much the same way as words are formed from the letters of the alphabet. Leibniz proposes to regard the first terms as constituting the first of a series of classes; the second class of the series consists of the first terms arranged in groups of two; the third class, of the first terms arranged in groups of three; and so on." (Parkinson [1966] p.xvii)

Parkinson also comments that in *Elements of a Calculus* (*Elementa Calculi*, 1686?) Leibniz maintains the intensional view of the subject-predicate relation, which he clearly distinguishes from the extensional view. Concepts do not depend on the existence of individuals. Thus Leibniz's work is clearly placed within the context of the logic of intensions and has nothing directly to do with the modern mathematical logic of extensions. On several occasions Leibniz did attempt to formalise and systematise aspects of reasoning – with quantifiers and so forth – so in the broadest sense he is a progenitor of the work the modern formalists. But *not in any precise way*. We may look to Leibniz for inspirations, but not for carbon copies of our later ideas.

Of the *Introduction ad Enclyclopeadiam Arcanum*, written probably between 1679 and 1686, Parkinson remarks that Leibniz held that "An analysis of concepts such that we can reach primitive concepts, i.e. those which are conceived in themselves, does not seem to be within human power." However, Leibniz still thought it worthwhile to search for concepts that may not be "absolutely the first" but are still "first for us". In the *Generales Inquisitores* Parkinson Leibniz continues to distinguish 'primitive' terms or particles (that are unanalysable) from those that are 'composite'. By particles here are meant grammatical particles such as 'in', 'with'. Particles may also be combined. (There is some kind of proto-Boolean algebra presented in this work.) Leibniz espouses the view that what to us are contingent truths are necessary from the perspective of God. He attempts to develop a symbolism for the 'propositional forms' and the universal affirmative, and so forth. In *Generales inquisitiones* Leibniz makes use "of a parallel line symbolism to represent the proposition from the intensional point of view." Leibniz also continued his attempts to represent propositions numerically – by means of the mathematical symbolism of ratios. Parkinson argues that at this stage Leibniz has probably not anticipated Boolean algebra. This work contains alternative attempts to symbolize (represent) the fundamental prepositional forms.

The universal characteristic of Leibniz is a species of concept writing. There is an attempt at formalism, but the formalism would not be akin to anything that we see in modern mathematical logic – it is a formalism of concepts not of extensions, which have no part to play in it. Regarding the universal characteristic itself, the following quotations from *De Arte Combinatoria*, [1666] are instructive: -

> 89. We have spoken of the art of complication of the sciences, i.e. of inventive logic, whose categories, as it were, would be formed by a table of terms of this sort. From what we have said there flows a corollary, or. Use XI: a universal writing, i.e. one which is intelligible to anyone who reads it, whatever language he knows." (Ar. I. 20, quoted Parkinson [1966] p. 10)
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> 90. … the signs will be a kind of alphabet … the whole of such writing will be made of geometrical figures, as it were, and of a kind of pictures – just as the ancient Egyptians did, and the Chinese do today. (Ar. i. I, 202, quoted Parkinson [1966] p. 11)

The universal characteristic would be a method of constructing definitions and is a form of *inventive logic* – not formal analytic logic. Leibniz envisages the symbolism of the logic in terms of hieroglyphics – that is, symbols denoted unambiguously and universally, independently of linguistic forms, *meanings* or *concepts*.

2 Primitive characteristics

Leibniz indicates that true primitive concepts (or characteristics) cannot be attained. But relatively, some are more primitive than others. The identification of those characteristics that are more primitive in our universal scheme of things is the work of *phenomenology*, and this is not the place to embark upon so huge a project. However, a few observations in this direction would be pertinent. Two fundamental primitive characteristics are: -

2.1 Fundamental primitive characteristics

1. Time

   Time is inconceivable without the possibility of counting successive moments in time. The continuum is time. As primitive characteristic time has two *insertions* (see below): -

   1.1 The now. The actual.

   1.2 The continuum. The possible.

2. Space.

   Extension. Space is the relation of part to whole; the part is bounded by the unbounded whole. As primitive characteristic space has two insertions: -

   2.1 The unbounded. The whole.

   2.2 The bounded. The part.

As indicated, each primitive characteristic, just like a jigsaw piece, has *insertions* that connect it to other pieces. *Insertions* are properties or concepts falling under concepts; they derive from the phenomenological fact that intuitions are given as manifolds of experience, that correlative concepts of the understanding never arise in isolation from each other. What makes primitive characteristics primitive is that they are not concepts that arise from the matching of their insertions to other primitives; they are not compounds. The following are combinations of primitives via their insertions.

2.2 Potential and actual infinite as characters

3. Potential infinite

   Space in time: the succession of all moments in time given as bounded moments (parts) to the unbounded whole (time as the continuum).

4. Actual infinite

   Time in space: the succession of all moments in space, given as a bounded totality of all parts of the continuum to space as a whole.

The following are insertions belonging to the potential infinite: -

   3.1 Counting number

   3.2 Mathematical induction

   3.3 Archimedean property = lack of bound

   3.4 Time as phenomenon (progression)

From these we derive the concept of formal number theory, or arithmetic. By contrast, analytic logic is the relation of bounded part to bounded whole – it is what one gets if one ignores the phenomenon that space as a whole is unbounded. Thus the primitive characteristic of analytic logic is not the same as that of formal number theory.

Set theory arises initially from the attempt to bind, complete and actualise the potential infinite. The concept of the actual infinite is combined with that of the potential infinite to create the bounded set of all "natural numbers"; this is the first uncountable ordinal that we symbolise by ω.

Set theory does not stop at ω, which is insufficient to analyse the continuum. Then the actual infinity, ω, is placed within an unbounded hierarchy of all infinities upon infinities – the so-called proper class of all sets. In this way we see that primitive characteristics are indestructible. It is not possible to bind the potential infinite and make it actual; likewise, it is not possible to embed the actual infinite within the potential infinite, which mathematical terms this means that *we need the Axiom of Completeness.*

In set theory we see that both the potential and actual infinite coexist and each is alternately "covered" by the other. In the concept ω the potential infinite *falls under*, or is *covered by*, the concept of the actual (infinite); and in the concept of the universal proper class of all sets the actual infinite *falls under* the concept of the potential or unbounded.

By the time of Newton it was thought that the actual infinite could be eliminated in favour of the potential infinite. The revolution of the 1890s crystallising around the work of Cantor reversed the relationship. That is why Kronecker rebelled. Now the time has come to achieve a new synthesis through the realisation that neither characteristic can be replaced by the other. We need both.

The logic of intensions, which is the dialectic, is in its most general aspect *the art of fitting the pieces together* – like some vast jigsaw. This analogy has been used before – for example, in Hesse's *The Glass Bead Game.* The dialectic is the attempt to *understand life.* The glass bead game is the *universal language of reason.* Synthetic logic is the universal characteristic of all combinations of all primitives. We see immediately that synthetic logic embraces a scope of reasoning that is far greater than any analytic logic could ever have. Among aspects of this universal language of reason we have the following: -

2.3 Principle of indestructibility

No primitive characteristic can ever be destroyed. Therefore, in no concept (characteristic) can the universal characteristic of the actual infinite replace or destroy the universal characteristic of the potential infinite.

2.4 Principle of transcendent consistency

Every primitive characteristic may be combined with every other. No two primitive characteristics are inconsistent. No primitive characteristic denotes an impossible state of the world. Reality must be experienced in conformity with the primitive characteristics. Each primitive characteristic has as many insertions as there are other primitive characteristics.

The Psyche (Kant's Self) embraces all primitive characteristics and their combinations and transcends them all.

3 Synthetic logic

Poincaré's thesis is that complete induction is a synthetic principle of reasoning. This means that it cannot be reduced to an analytic, formal logic constructed over a partition of space. Complete induction is creative in nature, since the conclusion *is not analytically contained* in the premises. The validity of Poincaré's thesis demonstrates the existence of a synthetic logic. Analytic logic is based *synthetically* on the analogy with spatial containment; as such it is revealed to rest in its entirety on a single fundamental synthetic intuition – just as formally it was said to rest upon the law of non-contradiction, while that law itself could only be maintained as a synthetic truth.

3.1 Principles of creative synthetic logic

1. <u>Principle of induction</u>
   The most obvious candidate. Finitely generated recursive variants of this argument can be embedded into set theory, but the full version quantifies over an uncountable set and cannot be so imbedded. This is the meaning of Gödel's theorem
2. <u>Pigeon-hole principle</u>
   The whole cannot be embedded into a strict part of itself: this is a fundamental geometric intuition. It is the basis of the theory of ratios. The notion of one-one correspondence is based upon it, and hence the theory of different cardinalities. The theory of rings requires it. The pigeon-hole argument, when it is invoked, adds to the conclusion more content than is contained in the premises and is *synthetic.*
3. <u>Prime number</u>
   This is a concept rather than an argument. There is something clearly synthetic about the concept of a number that is not divisible by other numbers. The concept of natural number is also a candidate.
4. <u>Contradiction</u>
   Argument from contradiction invokes a counter-factual lattice point that is subsequently proven *not to exist.* Although the derivation from this point may proceed *analytically*, proof by contradiction embraces a synthetic aspect.
5. <u>Continuity and completeness</u>
   This is the idea of invoking a limit to an infinite process in order to *fill in the points*, to *complete* the space, to provide for *continuity*, is a synthetic process. The Axiom of Completeness cannot be derived from other axioms; hence, it has a synthetic basis.

6. Topology

There is the very rich source of synthetic principles in such concepts as *betweenness*, which lend themselves to the *intuitive* axiom systems of geometry. The properties of the various two-dimensional topological surfaces are given synthetically. That the edges of a sheet can only be identified edges in a finite number of ways, and what the resultant global structure is in each case (projective sheet, sphere, torus, cylinder, Mobius band, Klein bottle, etc.) is a synthetic principle and not the analytic consequence of the subdivision of those spaces into parts.

7. Modus Ponens and chain logic

The (synthetic) basis of analytical logic is the analogy with containment in space. However, the soundness of the modus ponens rule and entailment *is not prima facie based on that spatial analogy*. That rule is based on the analogy of a ladder or chain, which is not the "picture" of containment. The ladder may point upwards or downwards, or indeed, there is nothing inherently impossible in a ladder for the links to form a circle or loop. It is the analogy with *walking*. Hence the "paradox of material implication" because material implication is the version of entailment that one obtains when one seeks to identify it with the analogy of spatial containment; intuitively the two conceptions "feel" very different. In chain logic based on the analogy with taking a step: *X* is joined to *Y*, *Y* is joined to *Z*, therefore, *X* must be joined to *Z*. This analogy says nothing whatsoever about the global geometry of the surface created by the joins.

4 Logic in general

4.1 The Dialectic

The logic of the dialectic is deliberately "formally" unsound. Progress is in the direction of a more complete interpretation of reality, for which interpretations that are formally inconsistent or revealed to be inconsistent with historically later interpretations is no impediment. The dialectic conflict between rationalism and empiricism is vital to progress in this sense. We progress from relative "ignorance" to relative "enlightenment". A theory is better than a previous theory if it is more inclusive. The synthesis must embrace the content of the thesis and antithesis. The "absolute" truth of a thesis is not is the criterion on which it is judged; human intellect is frail, and absolute truth is unattainable in any one man's lifetime. What makes a good theory is its inclusiveness – it builds upon the past and pushes us forward. Being "wrong" is not a greater sin than being dull.

4.2 The logic of axiom adoption

Specifically: the logic of the adoption of the completeness axiom. This is based on the analogy of *filling in space* or the *toy drum*. One has a *frame* and one wishes to draw a skein over the frame as tightly as possible. There is only one way to do this subject to that constraint. Therefore, there is no choice. An axiom is (perceived to be) necessary if it is perceived that there is no concrete alternative to adopting it. When we contemplate the Axiom of Completeness we have a "mere" choice: to adopt or not to adopt, for we have no negation of the Axiom. Concretely, there is no alternative. We have two structures: $\mathbb{Q}$ - the set of all rational numbers. L - the language of equations using coefficients from $\mathbb{Q}$. L is the language of $\mathbb{Q}$. There is a map from the language L to the structure $\mathbb{Q}$. This is a reflection principle. We wish to fit the language L perfectly to the structure $\mathbb{Q}$; that is *draw* the *skein* as tightly as possible over the *drum* $\mathbb{Q}$. When we attempt to do so, we discover gaps in the structure $\mathbb{Q}$. Therefore, to fit L perfectly to $\mathbb{Q}$ we must fill in the gaps. This means that we must extend $\mathbb{Q}$ to a structure $\mathbb{R}$ that is the completion of $\mathbb{Q}$. The relation of the Lebesgue to the Riemann integral exhibits a similar pattern. For the Lebesgue integral, the value of the integral must be continuous. It must be possible to step from one value to the "next". Therefore, the Riemann integral must be extended in only one way.

4.3 Chain logic

Next to argument by analogy [See Chap. 15, Sec. 9], chain logic is probably the most fundamental kind of logic there is. Precisely for that reason, the structure created is globally amorphous. It is the logic of concept space, and there is no uniform notion of distance measurement in regard to concepts; rather, we have an intuitive notion of nearness – one concept is felt to be close to another; or closer than some third concept; but we do not measure those gaps by means of a ruler. Mathematical induction is a species of chain logic. But it rests on a primary intuition *that there is an infinite chain*. The step of adding 1 always leads away from the point of origin and may be repeated indefinitely. Therefore, this does tell us something about the space globally; namely that it is capable of indefinite extension and is unbounded and directed (away from the origin). But this is amorphous. To add further structure we need to apply new analogies. If we permit only one dimension, then the chain becomes the scaffold of a line. If we add the rule of uniformity, that all steps are of the same size, then we get a coordinate system. In empirical reasoning, the connections of chain logic are causal laws. The chain is made by experience and sometimes formalised in mathematical language.

4.4 The logic of algorithms

The *logic of algorithms*, that is of formal systems, is not prima facie analytical logic either. It is a form of chain logic where the steps are such that a machine could effect them. The logic of algorithms makes no intrinsic reference to soundness in the analytic sense. The term "algorithm" is highly ambiguous. By algorithm we mean a process that can be effected by a formal system. By a formal system we mean a process that can be implemented on a Turing tape or equivalently by a Markov algorithm. For the logic of algorithms to be "philosophically interesting" it must make a prima facie attempt to represent mathematics. A machine can be programmed to fill in the tape in any fashion whatsoever; it may be connected to a finite input (external source) that results in any reassignment of 1s and 0s to the tape. Such a machine is useless as a model of mathematics, which by virtue of its connections is not like this. Therefore, to make the logic of algorithms into a model of mathematics, we must constrain algorithms to be sound. This creates a sub-domain of the logic of algorithms that is a model of analytic logic.

4.5 Logic of intensions

Two separate structures in the language *L* mean the same object (point) in the structure *S*. Concepts are not identical merely because their extensions are equivalent. Concepts represent perspectives or views of a structure. The same structure may be viewed from different vantage points. Information comes in many forms. But we have: -

1. Information about analytical connections in a structure *S*.
2. Information about equivalence or identity of perspectives.

The first gives rise to extensional, analytical logic; the second to the logic of intensions. The proof of Lagrange's theorem arises from a *change of perspective*. [See Chap. 12] We switch from the perspective of group axioms to the perspective of division. The identity of a Boolean lattice to a Boolean ring is also a change of perspective. The change is too complex to be viewed as an isomorphism; indeed, the two structures are *not isomorphic*, but they are identical nonetheless.[1]

[1] An algorithm can be written to translate from the Boolean algebra to the Boolean ring.